Study Guide: Glass Wall

written by Maria Andersen

Revision 2007.1. Written to accompany the book, *The Glass Wall,* by Frank Smith.

Name ________________________

<u>The Glass Wall: Introduction Study Questions (p.1-6)</u>

1. Give <u>your own</u> example of a statement in mathematical language.

2. Give <u>your own</u> example of a statement in natural language.

3. Why might it be difficult for a young child to understand that the opposite of "more" is "less"?

4. When a student reaches the point where mathematics seems "dense and impenetrable" what has happened? (describe it in your own words)

5. List the three basic parts of mathematics that are firmly situated in our heads.

    a.

    b.

    c.

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<u>The Glass Wall: Chapter 1 Study Questions (p.7-15)</u>

1. What about "natural language" makes it necessary for mathematics to have its own language?

2. Give <u>your own</u> example of a subject area that needs to have its own precise language separate from the natural language.

3. Describe two mathematical activities that you engage in personally without being aware that you are doing so. (do not use the book examples)

   a.

   b.

4. Now describe two pieces of mathematical knowledge that you have stored in your mind, but you have never put to practical use. (do not use the book examples)

   a.

   b.

5. Some people argue that mathematics is a "universal language" because so many aspects of the physical world can be described by mathematics. Why, according to Smith, is it not surprising that the physical world follows the rules of mathematics?

6. Where does the bulk of mathematical ideas exist?

7. Give <u>your own</u> examples of something that is discovered.

8. Give <u>your own</u> example of something that is invented.

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The Glass Wall: Chapter 2 Study Questions (p.16-22)

1. Below are the four unique and universal characteristics of human brains that make human accomplishments possible. Briefly explain each in your own words.

   a. SEEK -

   b. EXPECT -

   - Consistency -
   - Coherence -
   - Consensus -

   c. BRING -

   d. RESPOND -

2. When do you know that you have "internalized" mathematics?

3. Why is the ability to "categorize" an important one?

4. We are born with the two basic ideas of algebra. What are they? Give an example of each.

   a. ________________ Example:

   b. ________________ Example:

5. Give an example where we might…

    a. Recognize an innate pattern

    b. Complete a pattern

    c. Impose a pattern

6. A common criticism of students today is that they do not realize when a wrong answer to a problem is inappropriate (i.e.Answer: John's age is -13). What characteristic does Smith write about that could be used to describe this issue?

7. How does emotion affect our ability to learn?

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<u>The Glass Wall: Chapter 3 Study Questions (p.23-34)</u>

1. Approximately when was mathematics first expressed in a <u>written</u> form?

2. What does the phrase "to express numbers economically" mean?

3. How does natural language universally distinguish between "one" and "more than one"?

4. What is a "mass noun"? Give <u>your own</u> example that shows why we need them.

5. How are the meanings of these words different from the mathematical meaning when we use them in everyday contexts? Write <u>your own</u> sentence for each word to demonstrate its everyday context (different from mathematical context).

   "add"

   "minus"

   "difference"

   "negative"

   "multiply"

   "divide"

   "equals"

6. Think of another mathematical word with dual meanings (like in #9) and give an example sentence with the alternative meaning.

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The Glass Wall: Chapter 4 Study Questions (p.35-41)

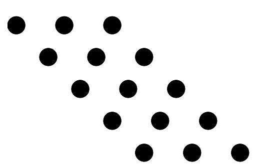
1. The author gives three reasons for why a number cannot be defined as a quantity of something. Look up the word “number” in a dictionary, and copy the definition (there will be many) that you think comes the closest to describing a number in the same way as the author.

2. What do we have to start doing with numbers in order to cross over from the natural language to the world of mathematics?

3. What is “subitizing”?

4. About how many objects can we subitize without using some calculation?

5. You ask a classmate to tell you how many dots are below. They tell you there are 15 dots. Describe a way they could have quickly determined that. Is this experiment an example of subitizing?



6. 20,000 is a pretty big number, but a salary of $20,000 is not actually all that much. Describe how you could try to help a child to understand the salary $20,000.

7. The author says that analogies don’t really help us to <u>understand</u> large numbers. Why not?

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The Glass Wall: Chapter 5 Study Questions (p.42-51)

1. When the theoretical shepherd used a pebble to "stand for" a sheep, this is an example of which innate concept of algebra that humans understand (described in Ch.2)? (one word)

2. You show the following picture to some children and explain that each dot represents a child standing in line. Then you ask which line has more children. What is likely to happen? Why? Who was the researcher who described this phenomenon?

   Line 1 ●●●●●●●●●●

   Line 2 ● ● ● ● ● ● ● ● ● ●

3. *Calculus* is the Greek word for _____________. Explain why this word might have been originally chosen to describe calculations in mathematics?

4. Use the dots below to demonstrate how to show a child the difference between the cardinal number 5 and the ordinal number 5.

   ● ● ● ● ●   ● ● ● ● ●

5. Why are the words "eleven" and "twelve" anomalies? What "words" would actually make more sense to describe 11 and 12?

6. In the decimal system, we have approximately 25 different number words, they are:

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<u>The Glass Wall: Chapter 6 Study Questions (p.52-60)</u>

1. To have useful **<u>written</u>** mathematics, two problems had to be solved, they were:

2. Name the following representation systems, decide whether the statement is true or false, then list the main problem with this system:

	Name of System	True or False?	Main Problem
$\text{IIIIIIII} \times \text{III} = \text{IIIIIIIIIIIIIIIIIIIIII}$			
$\text{LVI} + \text{LXII} = \text{CXIII}$			

3. Students sometimes inherit the idea that zero is written as $\varnothing$, as in the number $4\varnothing23$. However, $\varnothing$ really stands for the empty set, or "no solution." Use the following algebra problem to justify why it is important to correct this **<u>incorrect</u>** notation for zero early on.

$5x-8=3x-8$ Why should the notation be corrected?

$2x-8=-8$

$2x=\varnothing$

$x=\varnothing$

Is there a solution to this problem? Yes or no? ____ If yes, what is it? ____

4. One concept of negative numbers that Smith does not present (and one that makes it easier to explain the arithmetic involved), is that negative numbers represent the opposite of positive numbers. For example, -3 is the opposite of $+3$. So $-(-3)$ is the opposite of -3, which is ____. And in English, a double negative is positive. Consider the following example:

I failed the test. (one negative, this is a negative statement overall)
I didn't fail the test. (double negative, this is actually a positive statement, we can rewrite it …)
I passed the test. (again a positive statement)

Now write your own English double negative that you could use in the classroom.

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<u>The Glass Wall: Chapter 7 Study Questions (p.61-70)</u>

1. Numbers that refer to nothing more than the object they are attached to are said to be categorical or ___________ (a word we used in the textbook). A grammarian classifies such numbers as __________________.

2. There is a mistake in the book (oops Frank!). The numbers 1 to 9 do <u>not</u> distinguish among 99 players. It would be the numbers 0 to 9 that distinguish among 99 players (or 100 if you count 00 as a player). So… the numbers 1 to 9 actually distinguish between only 90 players. Why?

   How many players would be determined by using one letter (A-Z) and one number (1-9)? ______ How about one letter (A-Z) and one number (0-9)? _____

3. To be able to assign order, how many objects are required? _____ It is not the objects, but the __________________ that is assigned order.

4. Smith says that ordinal numbers can't be used for calculation, because they don't have to begin from a fixed point. Hmm…I think there are <u>some</u> cases where they CAN be used in calculation. Write down an example where you might make a **<u>calculation</u>** using a comparison of ORDINAL numbers.

5. Shows why successive numbers cannot be allocated more than once to each of these objects to be counted.

   ⊗ ⊗ ⊗ ⊗ ⊗

6. Show why the order in which the objects are numbered is irrelevant.

   ⊗ ⊗ ⊗ ⊗ ⊗ ⊗ ⊗ ⊗ ⊗ ⊗

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The Glass Wall: Chapter 8 Study Questions (p.71-82)

1. Why is the leap from counting to calculating so important? (use your own words)

2. What do the three C's represent?

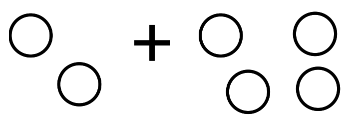
   What are the three C's?

   C

   C

   C

3. Demonstrate how a child that adds by "counting-on" might add the objects below.



   Why would it be dangerous to encourage a child to use "counting-on" for all their addition skills? (in your own words)

4. Are there actually 365 days in a year? Explain.

5. Why is there not an "official standard" in existence somewhere for the angle measure of 1°?

6. We encounter many units in our daily life that we actually don't understand. For example, estimate the number of calories in an apple ______. Estimate the number of calories in a Big Mac ______.

   Describe a calorie in your own words.

   Look up a calorie's exact definition in a textbook or dictionary.

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The Glass Wall: Chapter 9 Study Questions (p.83-91)

1. Summarize the difference between a “sign” and a “symbol.”

2. Prior to the Internet, the **sign** @ meant to add up a total involving multiplication. For example, 3@$2.50 would be 2.50+2.50+2.50= $7.50.

   What does the **symbol** @ indicate to us now?

3. What meaning might children attach to an “=” sign in elementary school?

4. What meaning might an algebra student attach to an “=” sign?

5. A student writes “$-4 > x > 8$” as the answer to a problem and tells you that this says that *x* is between -4 and 8. Why is “$-4 > x > 8$” not a correct mathematical statement?

6. What is the answer to $7\ 8$ according to Newton? _____

7. Sometimes in math, the same symbol represents two meanings. Describe what the “$-$” represents in both places where it occurs in $-3-5$.

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The Glass Wall: Chapter 10 Study Questions (p.92-101)

1. What fraction is most natural to us?

2. What were the two problems encountered as people began to explore the concept of fractions?

    a.

    b.

3. What are the two ways you could read "4 / 7"? Write them out in words.

4. ____________ and ___________ are pure magnitudes. A _________ is a relative magnitude.

5. Indicate whether the following statements are relative magnitudes or magnitudes:

    The house is 4.2 meters high. ______________
    The building is 4 times bigger than that house. ______________

    What does "*relative* magnitude" mean? (as opposed to just magnitude)

6. Why could it be problematic for a child to think that *multiplication makes something bigger and division makes it smaller*? Use a numerical example **for each** to illustrate your point.

7. How are rational decimals really fractions?

8. In the number 0.123, the author claims that the zero does not stand for anything, but what DOES it actually stand for?

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The Glass Wall: Chapter 11 Study Questions (p.102-111)

1. How was the study of geometry changed when it was transferred from physical measurements to paper?

2. Why is it difficult for us to imagine that a rectangle might represent a floor plan of a house?

3. We can only represent three-dimensional objects by using ________________.

4. Why is it not necessary to draw geometric figures accurately?

5. Who developed the coordinate system? __________________

6. What problem hindered early mathematicians and mapmakers from developing the idea of coordinates?

7. Look up the definition of a *function* in an algebra or precalculus book. What is it?

8. The notation for a function is ________. If f represents the rule "add five then multiply by three," then what is $f(2)$? _____ What is $f(a)$? _________

9. Is a line a curve? ______

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The Glass Wall: Chapter 12 Study Questions (p.112-121)

1. What does "calculating" mean?

2. What is the "brute-force" method of solving a math problem? ____________

3. How many mathematical facts must be learned before arithmetical calculation can begin comfortably? ________ Go to the Internet and see if you can find some fun ways to help children learn the basic addition and multiplication facts. You might try searching "multiplication songs" or "addition fun" etc. What did you find? Do you think it would help?

4. Why is it essential to have the "basics" memorized for learning how to do calculations? You might use an example to illustrate your answer.

5. No doubt you will have to face the issue "Calculator or No Calculator" sometime in your career. What are the arguments for learning calculations by hand?

   a)

   b)

   c)

8. What are the arguments for using a calculator?

9. Do calculators actually store tables of data? Explain.

The Glass Wall: Chapter 13 Study Questions (p.122-135)

1. If you want the kids in your classroom to "become mathematicians," then what kind of characteristics would you encourage them to have?

2. When you're grading papers, you check the answer to a problem and see that the child has the correct answer, so you mark the child's answer as correct. But if it turns out that the work was actually wrong and the answer was just coincidentally correct, what effect does that have on the child?

3. Why should you NOT teach "the alligator eats the bigger number" when you teach inequalities to students?

4. The fourth essential condition for learning mathematics is "Time". What happens to the students who fall behind? What are some ways you can let individual students work at a natural pace?

5. Why should you NOT teach that multiplication of two numbers results in a larger number?

6. Why is it harder for students to work with 16 than 36?

7. Many students (some of you included) simply panic when they see a "word problem." Often they don't even read the problem! What techniques can you use when a student comes to you for help and you suspect that the problem has not even been read?