Introduction: Group Questions

1. What is the "glass wall" according to the author?

2. When children are taught to count, what are we teaching them in a language sense?

3. The author uses music as an example of another realm with its own language. Describe an example that shows the difference between knowing the language of music and understanding the world of music at a deeper level (that is, from behind the glass wall).

4. Do you think that the real understanding of what the number words mean is achieved through sequential or global learning? Why?

5. Brainstorm what you think the "few basic concepts" that are already established in spoken language" are. (You might think about what a very young child might already know word-wise or what kind of concepts early man might have understood.)

Chapter 1: Group Questions

1. Why was the ability to write down mathematics so important to the development of the mathematical realm?

2. People can speak without understanding the rules of spelling and grammar. How is this similar to mathematics?

3. Why is the language of mathematics not like a natural language?

4. How would you define mathematics to a child?

5. Do numbers exist? Why or why not?

6. If you were to write a mathematical biography that described your <u>social</u> mathematical history, what relationships would you consider?

Chapter 2: Group Questions

1. Which of the characteristics of the human brain do you think are strongest in children and why?

2. Why is it important that our environment provides us with clues about our behaviors?

3. *"Patterns don't exist in the world; they exist in our head."* (p.21)
   Do you think we are the only creatures on this planet that have patterns existing in our head? How about other characteristics, like abstraction, generalization, categorizing, and relationships?

4. *"We have conventions for everything – including ways to be unconventional!"* (p.21)
   Come up with examples that support this statement. Do you think the culture you live in affects whether there are conventions for being unconventional?

5. *"Humans were bound… to magnify their strength, amplify their perceptual abilities,…"* (p.22) Smith goes on to say it's like "climbing up the rungs on a ladder, only, you can not go back down. Think about this in the context of scientific discovery. Once we see the possibility for an invention or possibility, do you think it is possible to stop it? List as many examples as you can of this observation.

Chapter 3: Group Questions

1. On p.27 there is an out of order sequence of days of the week. When you read it, does your brain try to "force" the words into the right order?

   Can you read the following passage? (don't try to hard)

   Mray had a ltitle lmab, its flcee was wihte as sonw.

   Why are you able to read it even though it's nonsense?

2. Why, according to the author, is it not enough for young children to simply be able to <u>recite</u> the numbers between 1 and 10?

3. Think of at least two more examples (besides the ones in the book) of a "chant" involving numbers.

4. List some of the "infant words" that demonstrate the following terms

   a. Categorization

   b. Plurality

   c. Quantity

   d. Sequencing

   e. Distance

   f. Size

   g. Speed

   h. Geometry

Chapter 5: Group Questions

1. There were two necessary developments in writing numbers to make mathematics really take off, what were they?

2. In a one-handed system of counting, how would you describe the number 23?

3. Even though a binary number system (which is base ___) is widely used in computing technology, it is unlikely that it will ever be used for practical calculations. Why?

4. Write the number 64 in each base system below:

Base 10	64
Base 9	
Base 8	
Base 7	
Base 6	
Base 5	
Base 4	
Base 3	
Base 2	

As the base decreases, what happens to the notation for the number?

Chapter 6: Group Questions

1. Give several examples of how to write 123 in Egyptian notation:

   _______________ _______________ _______________

   The Egyptian number system is **not** a positional system, while our modern decimal system **is** positional. Explain what it means for a number system to NOT be positional.

7. The number "zero" was widely rejected as nonsense because it was a number that "stood for nothing." **But zero does <u>not</u> stand for nothing.** What does the zero in 4023 stand for?

8. Here is a simple argument that can help you to explain to children that $0 \div 0$ is undefined (to really explain why, we would need to use a little bit of Calculus).

   $\frac{3}{3} =$ $\frac{2}{2} =$ $\frac{1}{1} =$ would lead you to believe that $\frac{0}{0} =$

   $\frac{0}{3} =$ $\frac{0}{2} =$ $\frac{0}{1} =$ would lead you to believe that $\frac{0}{0} =$

   So, following this argument, $\frac{0}{0}$ has to be undefined because…

9. Now let's find a clever way to explain **multiplication** rules for positives and negatives as they might occur in English. (this is a trick I learned from my 7th grade Algebra teacher)

   Here's the situation… Bridget loves to dance, so she goes to a school dance and waits to be asked to dance. Bridget has a huge crush on Sam, but she really, really doesn't like Mark.

   For Bridget, is dancing positive or negative? __________
   For Bridget, is Sam positive or negative? ___________ Mark? __________

Situation	**Dancing (+ or -)**	**Boy Involved (+ or -)**	**Overall situation (+ or -)**
Bridget dances with Sam.			
Bridget dances with Mark.			
Bridget does not dance with Sam.			
Bridget does not dance with Mark.			

The Glass Wall: Chapter 7 Group Questions (p.61-70)

1. Is the alphabet a positional system? Think carefully and justify your response.

2. Smith offers the idea that sometimes we can derive meaning from categorical numbers if they are given out in a systematic fashion. Think of a categorical number that has been assigned to you that tells someone something about you? (you might try comparing some identification numbers with your group members to find meaning in the numbers)

3. A child looks at five apples and counts out loud "one, two, seven, ten, twenty." How does this show that the child understands the complexities of **tallying**? (use your own words)

The Glass Wall: Chapter 8 Group Questions (p.71-82)

1. Express ten ways to get the number 9 using the operations $+, -, \times, \div, \sqrt{\ }$, or ^{2}.
   (use each operation at least once)

2. Get out a ruler, and do some measurements on one of your group members (this will also require some knowledge of converting between inches, feet, and yards)…

   Measure the distance from the tip of your thumb to the knuckle in inches.
   My knuckle measures _______ inches. Thus 1 "thumb" = ___ inches.

   Calculate: If 12 thumbs = 1 foot, then a foot should be _____ inches.
   Measure the length of your foot in inches. Actual foot length _______.

   Calculate: If 3 of your group member's feet measurements = 1 yard, then a yard would measure ______ inches. How many inches is a yard really? _______

3. Find some kind of "body measurement" for each of the following metric units:

   a. Something that is approximately 1 cm.

   b. Something that is approximately 1 mm.

   c. Something that is approximately 1 m.

4. Can everything be measured? Why or why not?

5. If scientists pressured politicians to create a metric calendar system (i.e. 10 "days" in a week, 10 "hours" in a day, etc.), what problems (besides a long-work week) do you think they would run into?

The Glass Wall: Chapter 9 Group Questions (p.83-91)

1. Write a mathematical expression with many nested sets of parentheses, then show how to process the expression?

   Write an English sentence with nested sets of parentheses, how do we process this sentence?

2. What is the difference between an unknown and a variable? See p.89.

   a. Write an example where *x* is an unknown.

   b. Now write an example where *x* is a variable.

3. To demonstrate what might be confusing about mathematical notation, what do each of the following mean or represent mathematically?

X	____	a. multiplication
x	____	b. a vector
$\times$	____	c. a set
$\overline{x}$	____	d. a mean
$\mathbf{x}$	____	e. a variable

4. Some letters get used in math and science over and over. What might these letters stand for in an application problem. Think of as many different representations as you can.

   v ____

   p ____

   c ____

   t ____

The Glass Wall: Chapter 10 Study Questions (p.92-101)

1. Find some way to explain why $\frac{1}{2} \div \frac{1}{3} = \frac{3}{2}$ <u>without</u> inverting the second fraction and multiplying.

2. In your own words, what are the two major differences between the whole number system and those numbers in between the whole numbers?

3. Okay, now for some REAL fun!!! Let's do decimals in base 5…

   First we better practice with base 5 a little. What are the digits in base 5? _________

   What does each place (base 10) stand for in the diagram below:

   ____ ____ $\underline{10's}$ ____ . ____ $\frac{1}{100}$ ____ ____ *(use fraction representation)*

   What does each place in base 5 stand for in the diagram below:

   ____ $\underline{25's}$ ____ ____ . ____ ____ ____ ____ *(use fraction representation)*

   Show why 241_{five} is the same as 71_{ten}.

   Show why 0.2_{five} is the same as 0.4_{ten}.

   Now show why 12.43_{five} is the same as 7.92_{ten}.

The Glass Wall: Chapter 11 Study Questions (p.102-111)

4. The development of the coordinate system led mathematicians to demonstrate what?

5. Why do we teach map-reading in elementary school?

The Glass Wall: Chapter 12 Study Questions (p.112-121)

1. Historically, math teachers have taught times tables up to 12's, why do you think that is?

2. Should a student desire to memorize everything in mathematics? Why or why not?

9. Based on what you have read in this chapter, what might be a good strategy regarding the type and quantity of math problems that you assign to the kids in your class?

10. Often textbook problems use "nice" numbers. What's the appeal of using "not-nice" numbers?

The Glass Wall: Chapter 13 Study Questions (p.122-135)

1. The first essential condition for learning mathematics is that the mathematics must be interesting and comprehensible. List some techniques that we have talked about in class that you can use in the classroom to meet this criteria.

2. Why should you do your best to not exhibit a "fear" of mathematics yourself?

3. Why is it not good to just teach "tricks" for learning math?

4. When you become a teacher you will be amazed at the naturally-occurring misconceptions that most students have. For example, in algebra, students have a hard fast misconception that $(x+y)^2$ is the same as x^2+y^2. Write down a mathematical misconception that you know you have trouble with.

5. It is beneficial to let students develop mathematical principles on their own and at their own pace, why can't children develop all their mathematical skills this way?