# General Categories of Instructional Practice

## Teacher-Directed Instruction

Description: Instruction in which the instructor is primarily communicating the mathematics to the students directly and in which the majority of interactions about the mathematics are between the teacher and the students. (NMAP Instructional Practices, Gersten et al., 2008)

- Clearly-prescribed instructional sequences
- Consistent focus on content objectives
- Emphasis on explanation
- Assessment and correction of errors
- Feedback to students
- Assignment and review
- Whole-group instruction

## Student-Centered Instruction

Description: It is primarily students who are doing the teaching of the mathematics and the majority of the interaction occurs between or among students. (NMAP Instructional Practices, Gersten et al., 2008)

- Emphasis on student responsibility and independence
- Acknowledgment of students' experiences, prior knowledge, interests, and motivations in the design of mathematics instruction
- Students thinking is central and students teach other students
- Teachers facilitate, encourage, and coach but do not explicitly instruct by showing and explaining how things work

# Mathematics Instructional Practices (MIPs)

What follows are ten mathematics instructional practices (MIPs) are meant to capture the nuances of the majority of mathematics instruction for the first two years of college mathematics. The use of technology for instruction is **not** its own category. Technology is illustrated as part of the implementation of various instructional strategies, but emphasis on technology itself is not an instructional strategy that can be easily summarized and is left to future exploration. The practices are also meant to describe the nuances of teaching face-to-face. The nuances of mathematics instructional practices for online courses will be left for future research. It is important to first establish a baseline for MIP that occurs in face-to-face instruction.

Remember, no instructor will fall distinctly into any one category. All of us will use a mix of each of these practices, to varying degrees. In the final instrument to measure instructional practice, you would be asked to what extent you use each strategy in your courses (at a specific level of mathematics) and you would respond on a scale. Likewise, many activities that you do in the classroom may span more than one category of instruction. The intent of the categories is to lend description to different nuances of instructional practice in mathematics.

## Lecture

Description: Teaching by giving a presentation on some subject for a time period longer than 20 minutes. This instructional method includes the exchange of questions and answers between the instructor and students. The key characteristic is that the students rarely interact with each other during this learning process.

Examples:

- The instructor presents a logical narrative on exponential functions using a whiteboard. The narrative includes definitions, example problems, and application problems. The instructor periodically asks if there are any questions about the material.
- The instructor presents a lesson on graphing lines using an overhead graphing calculator viewscreen to show students how changes to the algebraic function result in changes to the graph. The students follow along, each using their own graphing calculator and occasionally interject questions when they have a problem with the technology.
- The instructor uses PowerPoint and video from the Internet to present a lesson showing students how the path followed by a cannonball is modeled by a quadratic equation, and how to find that equation. Students with laptops click through the slides as they listen and watch the presentation.

## Collaborative Lecture

Description: Teaching by giving a series of short, focused lessons intermixed with student-centered activities that solidify the concepts of the lessons or serve to introduce the next short lesson (DeLong and Winter, 2002). The interaction during the activities is primarily between students.

Examples:

- The instructor begins class by showing the class a concept question that asks the students to choose which of four sets of data represent linear functions. After the students work to solve the problem in pairs, and share their results by responding with clickers, the instructor discusses the solution with the class and moves on to present a 10-minute lesson about one of the non-linear data sets, which can be represented with a quadratic function. This cycle of using a concept question followed by 10 minutes of lecturing continues for the whole class period.
- After giving a 15-minute lesson focused on solving a triangle using the Law of Sines, the instructor puts the students into groups and asks the students to solve two problems on their own. It turns out the instructor has purposely selected the second problem to have two possible answers and craftily pushes two of the groups towards the non-standard answer. When the groups present their findings to the rest of the class, this possibility of two solutions provides fodder to motivate the next 15-minute lesson on the possibility of more than one triangle for a set of data.

## Cooperative Learning

Description: Including class time for learning that engages students in working and learning together in small groups, typically with two to five members. Cooperative learning strategies are designed to engage students actively in the learning process through inquiry and discussions with their classmates (Rogers et al., 2001).

Examples:

- All of the students in the class find a partner and a spot at the whiteboards in the classroom. The instructor reads a factoring problem aloud and the students work together to solve the problem at the board. The students help each other within pairs and between pairs, asking questions and providing hints to each other. The instructor occasionally provides hints to pairs of students, but it is primarily students who are answering each others' questions. Every few minutes, the instructor directs one person from each pair to move to the right, and reads a new question for the new pair of students to solve together.
- The instructor poses the following question to an algebra class, "How do you find the least common denominator for any set of fractions?" Students are given two minutes to think about the problem on their own, and then they join a group to solve the problem. After 8 minutes, each group presents their solution to the rest of the class.
- Class is held in a room with eight computer stations. Students work together in groups of three to complete an activity about inverses using a spreadsheet program. One student is designated as the computer-specialist, one student has responsibility for writing the responses to turn in, and the third student will present the results of their experimentation to the rest of the class.

## Inquiry-based Learning

Definition: Designing and using activities where students learn new concepts by actively doing and reflecting on what they have done. The guiding principle is that instructors try not to talk in depth about a concept until students have had an opportunity to think about it first (Hastings, 2006).

Examples:

- Students use colored red and black counters to represent negative and positive integers. Students model the additions of signed numbers by matching up and removing pairs of red & black tiles until there are no more pairs. After several problems, each student proposes a "rule" for how to add integers of various types.
- Students to use spreadsheets or the data table on a graphing calculators to explore how a change in the function equation affects the data it produces. Students propose an explanation for what they see and then devise and conduct tests of their hypotheses.
- Students use the slider bars on an interactive online model to experiment with the effect of changing a coefficient on the graph of the function. Students work in teams to come up with a precise definition for how the coefficient affects the graph.

## Emphasis on Application Problems

Description: Including class time for students to solve problems based on data from real-world situations (present or past) problems that come from the partner disciplines of mathematics (e.g. Engineering, Chemistry, Biology, Physics, Economics, Business).

Examples:

- During class, students set up two equations to model the protein and carbohydrate content for two different foods, and then they solve the system of equations.
- Students download data from the instructor's calculator on the kinetics of a chemical reaction, find a curve to fit the data, and then make predictions based on the model they constructed.
- Students write mathematical equations to model the costs, revenues, and profits for a proposed business venture. Using the model, they find the break-even point for the venture and then present their findings at the end of class.

## Emphasis on Project-based Learning

Description: Designing and assigning project work that requires students to solve a non-standard problem that requires a longer period of time than problems that would typically be assigned for homework or in class. There is often a research component where students must actively seek data, background knowledge, or formulas. Often the students work on projects in pairs or small groups. The final result of a project might include a written paper or a presentation on the findings.

Examples:

- In an assignment that takes several weeks to complete, students are required to compile data from three different government databases, find functions to model the data, and then use the functions to make predictions about the future. The instructor reserves two days of class time for the students to present their findings to the class.
- Students in an algebra class are asked to design an experiment to test the assertion that the circumference of a circle is pi times the diameter. After writing a paragraph to describe how they are going to test the rule, and getting approval from the instructor, the student carries out several trials, records the data, analyzes it, and then writes up the results.
- Students work in groups to analyze statistics on the energy efficiency of one of the buildings on campus. Students have to determine (on their own) how to measure energy efficiency, gather the data, and analyze it. To present their findings, each group prepares a poster and then presents the results to the Supervisor of Buildings & Grounds.

## Emphasis on Multiple Representations

Description: Teaching by including multiple ways (e.g. graphs, diagrams, algebra, words, data, manipulatives) to represent mathematical ideas whenever possible. The rule-of-four (representing a function visually, algebraically, numerically, or with words) is an example of multiple representations.

Examples:

- The instructor describes the model of U.S. Postage rates (a nice piecewise function) in words. The students draw a graph of the function, the class contributes to constructing an algebraic formula for the piecewise function, and the instructor creates a blank table with inputs to which the students add the output results.
- The instructor shows the students how to multiply two binomials and the class works through several examples. Then the instructor uses a diagram of rectangles and squares to demonstrate the idea of multiplying two binomials and has the students each construct their own geometric example.
- The instructor puts the students in groups and gives each group a table of data. The students graph the data, find a function that fits the data, and then describe a situation that could be illustrated by the data.

## Emphasis on Communication Skills

Description: Providing opportunities for students to practice their ability to communicate mathematical and quantitative ideas using both written and oral communications.

**Examples:**

- At the beginning of each class, the instructor chooses two students to each present a selected homework problem. Each student presents their answer to the class, explaining each step to the class as they go.
- Ten minutes before the end of class, the instructor has the students write one paragraph about what they have learned in class that day. The writing assignments are turned in to the instructor, who chooses several paragraphs to copy (without student names), and then corrects the mathematical language, spelling, and grammar. The next day in class, each student receives a copy of the uncorrected student paragraphs to correct themselves. Then they compare their corrections with the instructors' corrections to learn from what they missed.
- In an algebra class, students are required to answer every application problem in a complete sentence that summarizes both the problem statement and solution.
- Students in a trigonometry class have to write their own application problem (and solution) based on some situation in their own life. The assignment is graded on the clarity and quality of the writing, and the accuracy of the mathematics.
- Students take a mathematical version of a "spelling test" where the instructor reads five problems aloud and the student writes the problems down. Then the instructor shows the students the answers, and students correct their own work to learn from their mistakes.

## Mastery Learning

Description: Designing summative assessment check-points into the instructional program where the student is tested on their mastery of a single topic (or subtopic). The instructor may coach students during class time or outside of class to help students who struggle with understanding the concepts while they are intensely focused on learning. Note that the student s do not receive partial credit for partially correct responses on mastery-based assessments.

Examples:

- The course is taught in a self-paced lab format. Students cannot continue to the next module in the course until they pass the previous module with an 80% on the end-of-module assessment. The instructor is present during lab time to answer questions, provide coaching, and motivate students to stay on track.
- Using the course management system Blackboard, an instructor sets up check point quizzes for students to complete after class each day. The quizzes are algorithmically generated, and students may take the quizzes multiple times, but they must answer 80% of the questions correctly before the score is recorded.
- At the end of the semester, students are each given an oral exam to assess their mastery of the subject. Each question the instructor asks is graded on a pass/fail basis and the oral exam is a major component of the course grade.
- Midway through the semester, students take a Gateway exam on logarithmic and exponential functions. Problems are graded correct or incorrect. The students must get at least an 80% to have a score recorded for the exam. They may retake the exam five times over a period of two weeks.

## Emphasis on Formative Assessment

Description: Making use of instructional strategies in the learning environment that assess where students are having problems so that students can learn more and learn better. (Gold, 1999)

Examples:

- During class, the instructor pauses in a lesson to project a concept-oriented true/false question on the board. The students answer the question by holding up one of two index card “bookmarks” that says either True or False. The instructor quickly judges the number of responses, then after a quick discussion (and certainty that the concept has been understood by most students), they proceed with the lesson.
- At the end of a class, the instructor asks students to write down their muddiest point from class on a scrap of paper, and collects the responses. The next day of class, the instructor begins by addressing a few of the issues that were collected on the previous day.
- Before each class, the students log in to the online course management system to answer three open-ended questions about the textbook readings assigned for that day. During the next class, the instructor uses a projector to show the class selected answers as a way to guide the lessons each class period.

## Notes about the development of the MIP list:

The first step in the process of developing the list of MIPs was to do an extensive review of the literature on instructional practices in mathematics. The literature review included, but **was not limited to**:

- Foundations for success: The final report of the National Math Advisory Panel (U.S. Department of Education, 2008)
- Beyond Crossroads: Implementing mathematics standards in the first two years of college (Blair et al., 2006)
- Professional standards for teaching mathematics (NCTM, 2001)
- CUPM Curriculum Guide (2004)
- CBMS Statistical Reports from 1990, 1995, 2000, and 2005
- Related articles from AMATYC Review (1994-2008)
- Related articles from New Directions for Teaching and Learning (2002-2009)
- Related articles from New Directions for Community Colleges (2002-2009)
- Related doctoral dissertations on instructional practices in mathematics (2001-2009)
- Related articles from MAA publications (2002-2009)
- Understanding Learning and Teaching (Prosser & Trigwell, 1999)
- First steps: The role of the 2-year college in the preparation of mathematics-intensive majors (Wood, 2001)
- Fundamental mathematics: Voices of the partner disciplines (Barker and Ganter, 2002)
- Learning to Teach and Teaching to Learn Mathematics (DeLong and Winter, 2002)
- Curriculum foundations project: Voices of the partner disciplines (Ganter and Barker, 2004)
- Achieving quantitative literacy: An urgent challenge for higher education (Steen, 2004)
- A Fresh Start for Collegiate Mathematics: Rethinking the Courses below Calculus (Hastings et al., 2006)
- Handheld graphing technology in secondary mathematics: Research findings and implications for classroom practice (Burrill et al., 2002)

**Informal email poll (March 21, 2009 – April 15, 2009):** I sent an open-ended request to list all the innovations and teaching movements that the instructor has seen in textbooks, teaching, or professional organizations during their math career. The question was asked via email to the AMATYC DistanceEd Listserv and about 20 instructors, researchers, and textbook authors with more than 20 years of experience in teaching mathematics and writing textbooks at the college level. There were approximately 20 detailed responses.

From the readings and email responses, I amassed a list of mathematics instructional practices that were mentioned multiple times in the literature. This list also included specific technologies and named-practices. The list needed to be narrowed down to be used as a component of the formal survey in my Knowledge-Attitude-Practice Research, but I did not want to bias the formal survey instrument by making arbitrary choices to narrow down the list. For this reason, I decided to conduct an informal poll of math instructors from K-12, community colleges, and 4-year institutions within my social network.

**Informal Poll #1 (April 28, 2009):** After narrowing the list to 20 possible "practices" for mathematics instruction, an informal poll was created to narrow down the list even further based on strategies that were more important to study. The practice of "lecturing" was not an option in this feedback poll since CBMS data shows that lecturing is still a widely used practice in mathematics. Volunteers were asked to try to choose 5 practices they felt were "very important to study right now", 5 practices that were "somewhat important to study right now", 5 practices they felt were "a little important to study right now" and 5 practices that were "not important to study right now" in an effort to narrow down the list. Volunteers were also asked to provide feedback about how to combine items and whether any were missing. A final question asked participants to weigh in on whether technology should be its own item or if it was a component the other practices. The poll was again distributed to my social network (formulated using Zoomerang) and posted on Twitter, the AMATYC Distance Ed Listserv, www.teachingcollegemath.com, and to approximately 30 instructors, researchers, and textbook authors. There were 50 respondents to the request for feedback, and the poll was closed within 24 hours. The results and responses are summarized below.

| **Practice** | **Not important to study right now** | **A little important to study right now** | **Somewhat important to study right now** | **Very important to study right now** | **Status in MIP** |
|---|---|---|---|---|---|
| Group learning | 8% | 26% | 24% | 42% | MIP |
| Hands-on manipulatives | 32% | 34% | 14% | 20% | Example |
| Mini lectures (breaking up class time) | 10% | 14% | 42% | 34% | MIP |
| Using projects to learn | 18% | 24% | 24% | 34% | MIP |
| Emphasis on writing and language | 22% | 35% | 27% | 16% | MIP |
| Peer instruction | 28% | 26% | 38% | 8% | Example |
| Visualization | 4% | 29% | 37% | 31% | MIP[1] |
| Graphing calculators | 22% | 29% | 29% | 20% | Example[3] |
| Rule-of-four | 42% | 23% | 25% | 10% | MIP[1] |
| Inquiry-based (discovery) learning | 8% | 31% | 31% | 31% | MIP |
| Real-world and application problems | 4% | 14% | 34% | 48% | MIP |
| Mastery learning strategies | 10% | 22% | 33% | 35% | MIP |
| Pre-reading strategies | 38% | 32% | 20% | 10% | Example |
| Using concept questions during class | 14% | 27% | 35% | 24% | Example |
| Clickers | 43% | 29% | 10% | 18% | Example |
| Online homework systems | 20% | 16% | 26% | 38% | Example[2] |
| Course management systems | 45% | 24% | 27% | 4% | Example |
| Math (CAS) software | 40% | 28% | 26% | 6% | Example |
| Spreadsheets | 47% | 29% | 14% | 10% | Example |
| Active boardwork by students | 22% | 29% | 24% | 24% | Example |
| **Total responses** | **236** | **258** | **240** | **230** | |

[1]Visualization and Rule-of-four were combined into Multiple Representations on the MIP

[2]Online homework systems are primarily used out of class to assess students summatively. A different study should be conducted to look at strategies that instructors use to motivate students to practice mathematics on their own, and online homework systems should definitely be included.

[3]Many comments were made that reflected the idea that technology is used to facilitate the instructional practice, and that it cannot be considered to **be** the instructional practice.

Based on the results of this poll, the field of mathematics instructional practices was narrowed down to ten practices. Reflecting on the general comments, use of specific technologies were removed from the list (with the intention to include them as examples to illustrate each instructional practice). One instructional practice, Assessment of Learning, was added to the list, based on feedback from the volunteers. The decision was made to move examples of online instruction in mathematics to future exploration.

At this point, I used the literature to write descriptions of each of the chosen instructional practices and several examples to illustrate the scope of the practice (including examples of technology use within the practice). Another informal poll was necessary to see if the chosen MIPs had clear descriptions, if the examples were indeed illustrative of the category, and whether any bias would appear because of the chosen wording.

**Informal Poll #2 (April 29, 2009):**

The poll was again distributed to my social network (formulated using Zoomerang) and posted on Twitter, the AMATYC Distance Ed Listserv, www.teachingcollegemath.com, and to approximately 30 instructors, researchers, and textbook authors. The purpose of the poll was to get feedback on the clarity and completeness of the descriptions. There were 46 respondents to the request for feedback, and the poll was closed within 24 hours. The results and responses are summarized below. The Ten Practices, descriptions, and reactions about wording were as follows:

| **Practice** | **Original Description** | **Very clear** | **Somewhat clear** | **Notes on Modifications made after feedback** |
|---|---|---|---|---|
| Lecturing | Teaching by giving a discourse on some subject for a time period longer than 20 minutes. This instructional method includes questions (and answers) between the instructor and students. | 79% | 7% | Several participants felt the need to strongly declare their beliefs about "lecturing" – the language of the MIPs was modified to remove "lecturing" and "lecture" and replace with words like narrative, lesson, and presentation. However, no good solution has presented itself to replace the title of this category. |
| Mini-lecturing | A series of short, focused lectures intermixed with student-centered activities that solidify the concepts of the lectures or serve to introduce the next small lecture. (DeLong and Winter, 2002) In student-centered activities, it is primarily the students who are doing the teaching of mathematics, and the interaction is primarily between students. | 61% | 37% | There was some confusion about the difference between mini-lecturing and lecturing. Lecturing and mini-lecturing (in their renamed formats) will appear on the same page to try to mitigate confusion. The category was renamed "Collaborative lecture." |
| Cooperative Learning | A form of active learning that engage students in working and learning together in small groups, typically with two to five members. Cooperative learning strategies are designed to engage students actively in the learning process through inquiry and discussions with their classmates. (Rogers et al., 2001) | 83% | 10% | There was some confusion about the difference between mini-lecturing and cooperative learning. Descriptions and examples were tweaked to try and clear up this confusion. |
| Multiple Representations | Using four ways to represent a function: visually, algebraically, numerically, or with words. This has been referred to as the rule-of-three, and later, the rule-of-four. | 59% | 33% | Feedback showed concern with the focus on functions in the description and examples. Description and examples were rewritten to be more general. |

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|---|---|---|---|---|
| Inquiry-based learning | Students learn by doing and reflecting on what they have done. The guiding principle is that instructors try not to talk in depth about a concept until students have had an opportunity to think about it first. (Hastings, 2006) | 70% | 20% | Description was tweaked slightly to make the distinction between cooperative learning and inquiry-based learning more clear. |
| Emphasis on Application Problems | Using problems based on data from real-world situations (present or past) or the incorporation of problems that come from the partner disciplines of mathematics (e.g. Engineering, Chemistry, Biology, Physics, Economics, Business). | 92% | 5% | To clear up the distinction between this item and project work, they will be placed on the same page. |
| Mastery Learning | The goal of mastery-based learning is for students to master a single topic (or subtopic) before moving on. | 82% | 18% | There was concern that the description described an outcome, and not a practice. The description was rewritten as a practice. |
| Project-based Learning | Project work requires students to work on solving a non-standard problem that requires a longer period of time than standard problems. | 65% | 25% | Feedback about the use of "non-standard problem" led to a more detailed description. Application Problems and Project-based learning should be placed on the same page to avoid confusion. Presentations are now included in the examples. |
| Emphasis on communication skills | Students practice their ability to communicate mathematical and quantitative ideas with both written and oral communications. | 72% | 18% | Examples were expanded to include oral communication (instead of just written). |
| Emphasis on assessment | Instructional strategies used in the classroom or learning environment that assess where students are having problems so that students can learn more and learn better. (Gold, 1999) | 69% | 21% | Title was changed to "Emphasis on Formative Assessment." Wording of examples was cleared up. |

Some other general changes were made to the organization of the MIPs, the titles of the practices, and the use of gender in the examples (although a concerted effort was made to use "he" and "she" in the examples, feedback comments indicated that using "the instructor" would be a better idea). Descriptions were carefully reworded to focus on the actions of the instructor to carry out the practices. The final result of this exploration is the list of Mathematics Instructional Practices that you now see on pages 2-6 of this document.